IT'S TIME FOR AN UPDATE – EXTREME WAVES AND DIRECTIONAL DISTRIBUTIONS ALONG THE NEW SOUTH WALES COASTLINE

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Introduction

The NSW offshore Waverider buoy network currently comprises seven Datawell Directional Waverider buoys which telemeter wave data to onshore recording stations. The near-real time wave data is collected by Manly Hydraulics Laboratory for the NSW Office of Environment and Heritage. The locations of the buoys are presented in Figure 1. The NSW offshore Waverider buoy network has collected over 250 station years of data since the first station was established in 1974. Four stations now have a record length of over 39 years and directional wave data is available for over 25 years off Sydney. The network represents one of the world's most comprehensive direct wave measurement datasets and is an irreplaceable asset of the NSW Government that continues to grow in value with increasing records.

Figure 1 – New South Wales Waverider Buoy Stations

The dates that each Waverider buoy station was commissioned and the date that each was upgraded with a Directional Waverider buoy is presented in Table 1.

Table 1 – NSW Waverider Buoy Station Data Availability – June 2017

Waverider Station	Date Station Commissioned	Directional Buoy Deployed	Record Length (Years)
Byron Bay	14-Oct-1976	26-Oct-1999	40.7
Coffs Harbour	26-May-1976	14-Feb-2012	41.1
Crowdy Head	10-Oct-1985	19-Aug-2011	31.7
Sydney	17-Jul-1987	03-Mar-1992	30.0
Port Kembla	07-Feb-1974	20-Jun-2012	43.4
Batemans Bay	27-May-1986	23-Feb-2001	31.1
Eden	08-Feb-1978	16-Dec-2011	39.4

Extreme Value Analysis

Preamble

An Extreme Value Analysis (EVA) is a statistical analysis dealing with the extreme deviations from the median of probability distributions to determine the probability of events that are more extreme than previously observed events.

An EVA utilising all NSW Waverider buoy data available from the MHL wave database to December 2009 was undertaken by the University of New South Wales Water Research Laboratory (WRL 2010). The EVA results presented in this paper have been derived using a methodology similar to that documented in WRL (2010). Storm events were extracted from the MHL wave database described above based on storm intensity and duration. Statistical analysis adopting EVA techniques was then undertaken to determine the Average Return Intervals (ARI) of storm wave events.

Storm Definition

Selection of what defines a storm is a crucial step in the EVA. The peaks-over-threshold method or the annual maxima method are typically applied for such analyses. As recommended in Goda (2000), the peaks-over-threshold method was adopted to maximise the number of events and improve confidence (reduce confidence intervals).

For this analysis, a storm event is defined as an event for the EVA when the significant wave height (H_s) exceeds 3 m for at least one hour. The significant wave height is defined as the mean wave height (trough to crest) of the highest third of the waves in a given recording period and is used extensively in coastal engineering.

For longer duration events (i.e. 3-hours, 6-hours, 12-hours or 24-hours), the maximum H_s exceeded for the duration of the storm was estimated by calculating the maximum of the “rolling minimum” (similar to a “rolling average” method). For example, the maximum value of H_s that is exceeded by 6 consecutive records during a storm event was applied for a 6-hour duration storm. This method was applied to the hourly data only and did not consider the 6 hourly data available prior to 1984.

In contrast to the methodology applied by WRL (2010), long duration events with lower wave heights (2.0 m < H_s < 3.0 m) were not included in this analysis. However, during some storm events, H_s may drop below the 3 m threshold for a short time before again exceeding the 3 m threshold. In this situation, if the time between such events was less than 24 hours and the storm event was clearly the result of an individual wave generating weather system, the two episodes were considered as a single storm to obtain statistical independence.

Furthermore, WRL (2010) defined the “n”-hour duration wave height as the value exceeded “n” times over the entire storm. For example, the 6-hour duration would be the 6th largest record during the storm. Using this method allows the top 6 wave height values to be independently selected from different times during the storm. This appears therefore not as representative of a 6-hour continuous duration as the use of the “rolling minimum” adopted in this study.

Methodology

The methodology applied for the EVA comprised the following steps:

- Storm event identification
- Probability distribution function fitting
- Goodness of fit estimation
- Determination of confidence interval.

The process applied to undertake the EVA and associated results are described below.

Storm Event Identification

All storm events that occurred until 30 June 2017 (only hourly since 1984-85) were extracted from each Waverider buoy station data set using an R Script for the selected storm duration as per storm definition presented above. Once identified, the storms were listed and ranked from largest to smallest peak H_s recorded during each storm event.

Probability Distribution Function Fitting

A probability distribution function was then fitted on the storm events. The Fisher-Tippett type I (FT-I or Gumbel) and Weibull distributions were applied for each buoy as they were identified as best fit distributions by Goda (1988) and You (2007). The two candidate distributions are:

FT-I Distribution Equation $F(x) = exp\left[-exp\left(-\frac{x-B}{A}\right)\right]$

Weibull Distribution Equation $F(x) = 1 - exp\left[-\left(\frac{x-B}{A}\right)^k\right]$

where $F(x)$ = distribution function
A = scale parameter
B = location parameter
k = shape parameter

The plotting position proposed by Goda (1988) was applied to the candidate distributions. This plotting position is:

$$F(m) = 1 - \frac{m - \alpha}{N + \beta}$$

where $\mathrm{F(m)}$ = expected probability of the m[th] ordered variates
N = number of storm events listed
α = constant given as 0.44 for the FT-I distribution and $\left(0.2 + \frac{0.27}{\mathrm{k}^{0.5}}\right)$ for the Weibull Distribution
β = Constant given as 0.12 for the FT-I distribution and $\left(0.2 + \frac{0.23}{\mathrm{k}^{0.5}}\right)$ for the Weibull Distribution

A reduced variate is then introduced for each candidate distribution to allow the determination of the scale, location and shape parameters by applying appropriate fitting methods. The reduced variate are:

FT-I Distribution Equation $\quad \mathrm{X} = -\ln[-\ln(\mathrm{F(m)})]$

Weibull Distribution Equation $\quad \mathrm{X} = [-\ln(1 - \mathrm{F(m)})]^{1/\mathrm{k}}$

The least squares method was applied for the fitting as this method was deemed more appropriate by Goda (2000) and You (2007) than the other methods.

The scale and location parameters (A and B) can directly be determined by applying a linear equation as follows:

$$\mathrm{H} = \mathrm{AX} + \mathrm{B}$$

The coefficient of regression (R^2) and the sum of the squares of the error (SSE) were evaluated as follows:

Coefficient of Regression: $$R^2 = \left\{\frac{N\sum_{i=1}^{N} H_i X_i - \sum_{i=1}^{N} X_i \sum_{i=1}^{N} H_i}{\sqrt{\left[N\sum_{i=1}^{N} H_i^2 - \left(\sum_{i=1}^{N} H_i\right)^2\right]\left[N\sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i\right)^2\right]}}\right\}^2$$

Sum of squares of error: $$SSE(H) = \sum_{i=1}^{N}(H_i - H)^2$$

where H_i = i[th] peak storm wave height
H = equivalent value evaluated by $H = AX + B$

The shape parameter k was estimated using the optimisation method developed by You (2007) as part of an extended least-squares method. This method consists of a goal seek analysis to determine k such as $|W-1|^{0.5} \le 0.01$ with:

$$W = \frac{\sum_{i=1}^{N}(H_i - \bar{H})(X_i - \bar{X})}{\sum_{i=1}^{N}(X_i - \bar{X})^2} \times \frac{\sum_{i=1}^{N}(X_i^* - \bar{X}^*)\,(X_i - \bar{X})}{\sum_{i=1}^{N}\left(X_i^* - \bar{X}^*\right)(H_i - \bar{H})} = 1$$

where $X_i^* = X_i ln[-ln(1 - F(m))]$

Goodness of Fit Estimation

Once all parameters are estimated, it is possible to compare the FT-I and Weibull distribution for each Waverider buoy dataset using R^2 and SSE. Results for each waverider buoy and different storm durations are presented in Table 2 and Table 3. It can be observed that for all durations and Waverider buoy storm datasets, the Weibull Distribution provides a better fit than the FT-I distribution.

Table 2 – Comparison of Goodness of Fit at each buoy for different durations between FT-I and Weibull distributions based on Sum of Square Errors SSE

Storm Duration (hours)	Sum of Square Errors (SSE)						
	Byron Bay	Coffs Harbour	Crowdy Head	Sydney	Port Kembla	Batemans Bay	Eden
1 (FT-I)	7.930	12.155	10.464	16.093	15.786	6.095	20.563
1 (Weibull)	1.515	1.841	2.552	1.463	1.617	0.347	1.847
3 (FT-I)	5.590	8.772	7.420	12.996	10.179	3.712	14.358
3 (Weibull)	0.975	1.517	1.739	1.356	0.611	0.333	3.077
6 (FT-I)	5.263	7.377	6.561	9.470	8.311	2.568	13.279
6 (Weibull)	1.394	0.864	1.240	0.788	0.717	0.326	4.340
12 (FT-I)	3.886	5.140	5.332	7.305	6.398	2.415	6.887
12 (Weibull)	1.148	0.926	1.124	0.512	0.621	0.446	2.469
24 (FT-I)	2.219	3.877	3.052	4.032	3.616	2.058	3.621
24 (Weibull)	0.275	0.714	0.714	0.405	0.189	0.601	1.289

Table 3 – Comparison of Goodness of Fit at each buoy for different durations between FT-I and Weibull distributions based on Coefficient of Regression R^2

Storm Duration (hours)	Regression Coefficient (R^2)						
	Byron Bay	Coffs Harbour	Crowdy Head	Sydney	Port Kembla	Batemans Bay	Eden
1 (FT-I)	0.975	0.965	0.968	0.969	0.965	0.970	0.956
1 (Weibull)	0.995	0.995	0.992	0.997	0.996	0.998	0.996
3 (FT-I)	0.974	0.962	0.969	0.968	0.968	0.973	0.956
3 (Weibull)	0.996	0.993	0.993	0.997	0.998	0.998	0.991
6 (FT-I)	0.967	0.958	0.966	0.969	0.964	0.974	0.948
6 (Weibull)	0.991	0.995	0.994	0.997	0.997	0.997	0.983
12 (FT-I)	0.959	0.953	0.958	0.961	0.954	0.962	0.953
12 (Weibull)	0.988	0.992	0.991	0.997	0.995	0.993	0.983
24 (FT-I)	0.947	0.928	0.946	0.948	0.930	0.926	0.943
24 (Weibull)	0.993	0.987	0.987	0.995	0.996	0.978	0.980

The graphical observations also confirm that the Weibull Distribution has a better fit than the FT-I distribution as illustrated in Figure 2. The graphs presented show the graphical fit of the Weibull Distribution (blue line) and the FT-I distribution (red line) in comparison to the extreme storms (black circles) for the 1-hour duration storms at each Waverider buoy. Similar observations were made for the other storm durations. The black circles were obtained using the following Annual Recurrence Interval (ARI) and return value (HR) formulas. For these reasons, the Weibull Distribution was adopted for the EVA for the seven Waverider buoy datasets.

Average Recurrence Interval $$ARI = \frac{1}{\lambda[1 - F(x)]}$$

Return Value $$H_R = F^{-1}\left(1 - \frac{1}{\lambda ARI}\right)$$

Confidence Interval

The 90% confidence interval (CI) is equal to 1.65 times the standard error of return wave heights for the extreme wave data $\sigma(H_R)$. According to Goda (1988), this standard error can be estimated as:

$$\sigma(H_R) = \sigma_z \sigma_x$$

Where σ_x is the sample standard deviation of the significant wave heights of the extreme storms and σ_z is the standard error of the return value obtained using the following equation.

$$\sigma_z = \frac{[1.0 + a(H_R - c + \varepsilon ln\nu)^2]^{0.5}}{\sqrt{N}}$$

with

$$a = a_1 \exp[a_2 N^{-1.3} + \kappa (ln\nu)^{0.5}]$$

and the parameters a_1, a_2, κ, c and ε determined from Table 4.

Table 4 – Coefficient of empirical formulas for standard deviation of return values when the true distribution is known (Goda, 1988)

Distribution	a_1	a_2	κ	c	ε	ν
FT-I	0.64	9.0	0.93	0.0	1.33	1.0
Weibull (k=0.75)	1.65	11.4	-0.63	0.0	1.15	1.0
Weibull (k=1.00)	1.92	11.4	0.00	0.3	0.90	1.0
Weibull (k=1.40)	2.05	11.4	0.69	0.4	0.72	1.0
Weibull (k=2.00)	2.24	11.4	1.34	0.5	0.54	1.0

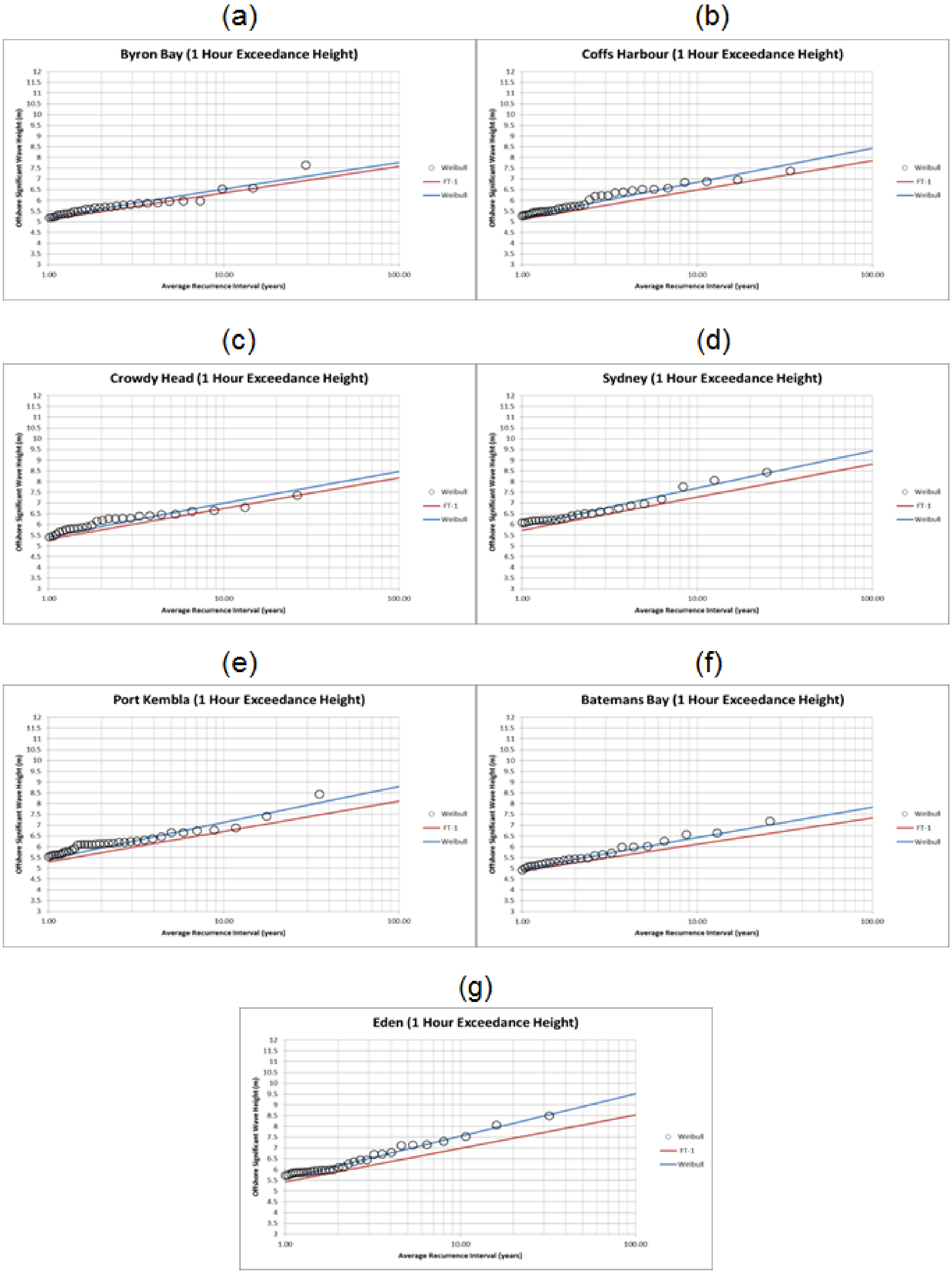

Figure 2 – Graphical comparison of FT-I and Weibull distribution for each Waverider buoy station

Results

The resulting significant wave heights (H_s) and confidence intervals (CI) for the different Waverider buoy stations and various storm durations are presented in Table 5 to Table 11. The graphical illustrations of the results are presented in Figure 3.

Table 5 – Extreme Wave Analysis Results for Byron Bay

ARI (yr)	Extreme Wave Analysis Results per Durations									
	1 hour		3 hours		6 hours		12 hours		24 hours	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
1	5.3	0.2	4.9	0.1	4.6	0.1	4.2	0.1	3.7	0.1
2	5.7	0.2	5.2	0.2	4.9	0.1	4.5	0.1	3.9	0.1
5	6.2	0.2	5.7	0.2	5.3	0.2	4.8	0.2	4.2	0.2
10	6.6	0.2	6.0	0.2	5.6	0.2	5.0	0.2	4.5	0.2
20	7.0	0.3	6.3	0.2	5.8	0.2	5.3	0.2	4.7	0.2
50	7.5	0.3	6.7	0.3	6.2	0.2	5.5	0.2	4.9	0.2
100	7.8	0.3	7.0	0.3	6.4	0.3	5.7	0.2	5.1	0.2

Table 6 – Extreme Wave Analysis Results for Coffs Harbour

ARI (yr)	Extreme Wave Analysis Results per Durations									
	1 hour		3 hours		6 hours		12 hours		24 hours	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
1	5.3	0.2	4.9	0.2	4.6	0.2	4.2	0.1	3.7	0.1
2	5.8	0.2	5.4	0.2	5.0	0.2	4.6	0.2	4.0	0.1
5	6.5	0.3	5.9	0.3	5.6	0.2	5.0	0.2	4.3	0.2
10	7.0	0.3	6.4	0.3	5.9	0.3	5.3	0.2	4.6	0.2
20	7.5	0.4	6.8	0.3	6.3	0.3	5.6	0.3	4.8	0.2
50	8.1	0.4	7.3	0.4	6.8	0.4	6.0	0.3	5.1	0.2
100	8.6	0.5	7.7	0.4	7.2	0.4	6.3	0.3	5.3	0.3

Table 7 – Extreme Wave Analysis Results for Crowdy Head

ARI (yr)	Extreme Wave Analysis Results per Durations									
	1 hour		3 hours		6 hours		12 hours		24 hours	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
1	5.4	0.2	5.0	0.2	4.7	0.2	4.4	0.1	3.8	0.1
2	5.9	0.2	5.4	0.2	5.1	0.2	4.7	0.2	4.1	0.1
5	6.5	0.3	6.0	0.3	5.6	0.2	5.2	0.2	4.4	0.2
10	7.0	0.3	6.4	0.3	6.0	0.3	5.5	0.2	4.7	0.2
20	7.4	0.3	6.8	0.3	6.4	0.3	5.9	0.3	4.9	0.2
50	8.0	0.4	7.3	0.4	6.8	0.3	6.3	0.3	5.1	0.2
100	8.4	0.4	7.7	0.4	7.2	0.4	6.5	0.3	5.3	0.3

Table 8 – Extreme Wave Analysis Results for Sydney

ARI (yr)	Extreme Wave Analysis Results per Durations									
	1 hour		3 hours		6 hours		12 hours		24 hours	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
1	5.8	0.2	5.4	0.2	5.1	0.2	4.6	0.2	3.9	0.1
2	6.4	0.2	6.0	0.2	5.6	0.2	5.0	0.2	4.3	0.2
5	7.1	0.3	6.6	0.3	6.2	0.3	5.5	0.2	4.6	0.2
10	7.6	0.3	7.1	0.3	6.7	0.3	5.8	0.3	4.9	0.2
20	8.2	0.4	7.6	0.4	7.1	0.4	6.2	0.3	5.2	0.3
50	8.9	0.4	8.3	0.4	7.7	0.4	6.6	0.3	5.5	0.3
100	9.4	0.5	8.8	0.5	8.2	0.5	6.9	0.4	5.7	0.3

Table 9 – Extreme Wave Analysis Results for Port Kembla

ARI (yr)	Extreme Wave Analysis Results per Durations									
	1 hour		3 hours		6 hour		12 hours		24 hour	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
1	5.5	0.2	5.1	0.2	4.8	0.2	4.4	0.1	3.7	0.1
2	6.0	0.2	5.6	0.2	5.2	0.2	4.7	0.2	4.0	0.1
5	6.7	0.3	6.2	0.2	5.7	0.2	5.1	0.2	4.3	0.1
10	7.1	0.3	6.6	0.3	6.1	0.3	5.5	0.2	4.5	0.2
20	7.6	0.3	7.0	0.3	6.5	0.3	5.7	0.2	4.7	0.2
50	8.3	0.4	7.5	0.4	6.9	0.3	6.1	0.3	4.9	0.2
100	8.7	0.4	8.0	0.4	7.3	0.3	6.4	0.3	5.1	0.2

Table 10 – Extreme Wave Analysis Results for Batemans Bay

ARI (yr)	Extreme Wave Analysis Results per Durations									
	1 hour		3 hours		6 hour		12 hours		24 hour	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
1	4.9	0.2	4.6	0.2	4.3	0.1	4.0	0.1	3.4	0.1
2	5.4	0.2	5.0	0.2	4.7	0.2	4.3	0.2	3.7	0.2
5	6.0	0.3	5.5	0.2	5.1	0.2	4.7	0.2	4.1	0.2
10	6.4	0.3	5.8	0.3	5.4	0.2	5.0	0.2	4.3	0.2
20	6.8	0.4	6.2	0.3	5.7	0.3	5.3	0.2	4.6	0.3
50	7.4	0.4	6.6	0.4	6.1	0.3	5.6	0.3	4.8	0.3
100	7.8	0.4	7.0	0.4	6.4	0.3	5.8	0.3	5.0	0.3

Table 11 – Extreme Wave Analysis Results for Eden

ARI (yr)	Extreme Wave Analysis Results per Durations									
	1 hour		3 hours		6 hours		12 hours		24 hours	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
1	5.7	0.2	5.2	0.2	4.8	0.2	4.4	0.2	3.7	0.1
2	6.3	0.3	5.7	0.2	5.3	0.2	4.8	0.2	4.0	0.2
5	7.1	0.4	6.4	0.3	5.9	0.3	5.3	0.2	4.4	0.2
10	7.7	0.4	6.9	0.4	6.3	0.3	5.6	0.3	4.7	0.2
20	8.3	0.5	7.4	0.4	6.7	0.3	5.9	0.3	5.0	0.3
50	9.1	0.5	8.1	0.5	7.3	0.4	6.3	0.3	5.3	0.3
100	9.6	0.6	8.6	0.5	7.7	0.4	6.6	0.3	5.5	0.3

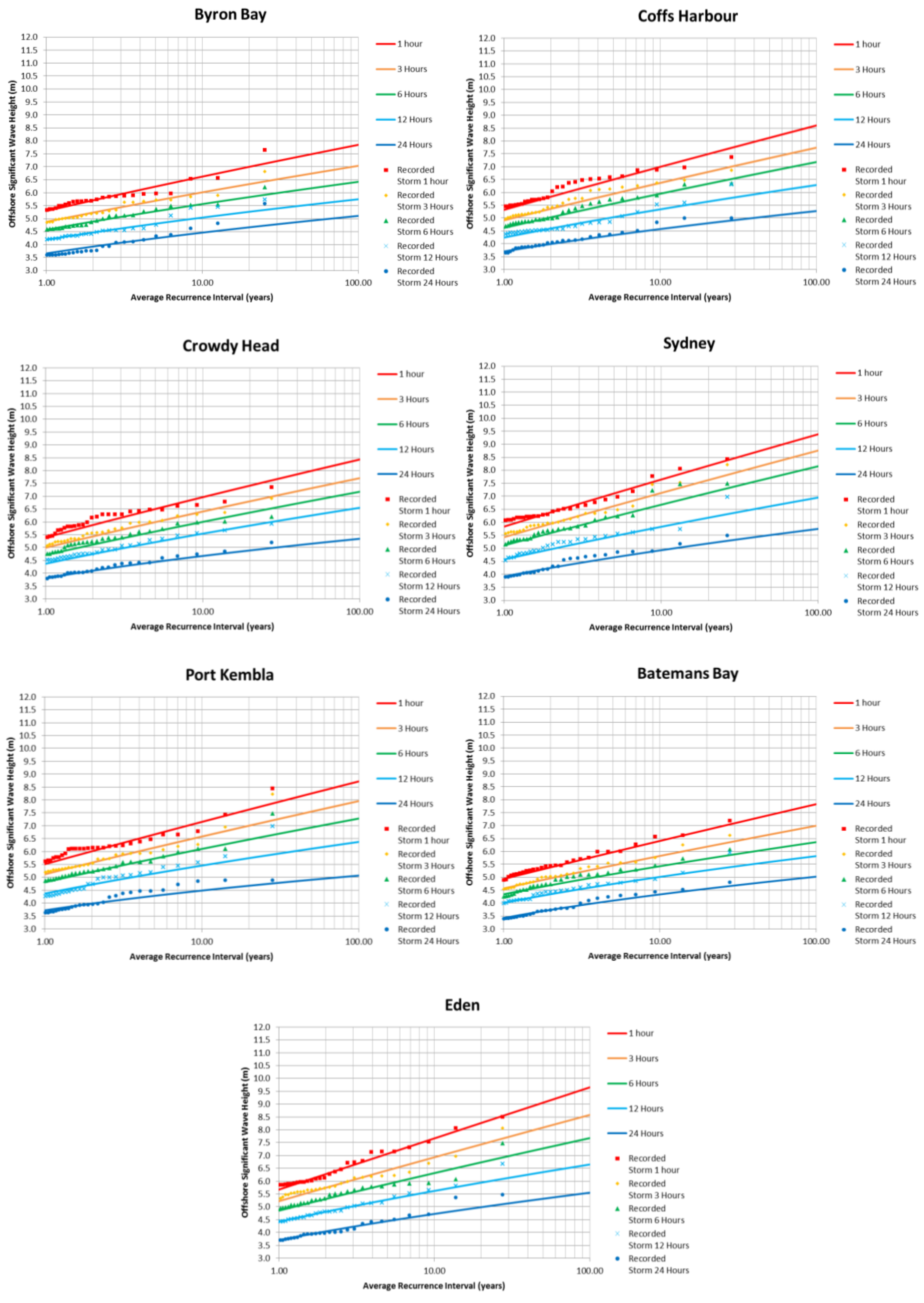

Figure 3 – Significant wave height extreme wave analysis results for each Waverider buoy station

Comparison with Previous Analysis

Direct comparison of the current method with the one applied by WRL (2010) was completed by analysing the data up to December 2009. Results of the analysis showed similar results between the two methods with differences in value up to ±0.2m for duration up to 12 hours. For a 24-hour duration, the differences increase to approximately ±0.5m. The analysis was then undertaken over the entire period of record up to 30 June 2017.

Results of the extreme value analysis for various ARI and their associated 90% confidence interval (CI) were compared to the results documented in WRL (2010). The comparison for the Sydney Waverider buoy is presented in Table 12. The results in this paper are comparable to those of WRL with some increases in H_s due to the occurrence of several major storms over the last few years. The longer storm durations (12 and 24 hours) have a slightly lower value than WRL (2010) due to two main factors:

- Given the use of a “rolling minimum” instead of the ranking of values, the values are expected to be lower;
- Consecutive sets of values including missing data were discarded to avoid misleading calculations when a large wave height value is followed by a long gap with missing data resulting in the 3, 6, 12 and 24-hour duration wave heights having the same value as the 1-hour record directly before the missing data.

Table 12 – Extreme Wave Analysis Results Comparison for Sydney Station

ARI (yr)	Extreme Wave Analysis Results per Durations (WRL, 2010)									
	1 hour		3 hours		6 hours		12 hours		24 hours	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
1	5.9	0.2	5.5	0.2	5.2	0.2	4.8	0.2	4.2	0.2
2	6.4	0.3	6.0	0.3	5.6	0.2	5.2	0.2	4.6	0.2
5	7.0	0.3	6.6	0.3	6.2	0.3	5.8	0.3	5.0	0.2
10	7.5	0.4	7.1	0.4	6.6	0.3	6.2	0.3	5.3	0.3
20	8.0	0.4	7.5	0.4	7.0	0.3	6.5	0.3	5.7	0.3
50	8.6	0.5	8.1	0.4	7.6	0.4	7.0	0.3	6.1	0.3
100	9.0	0.5	8.6	0.5	8.0	0.4	7.3	0.4	6.4	0.3
ARI (yr)	**Extreme Wave Analysis Results per Durations (MHL, 2017)**									
	1 hour		**3 hours**		**6 hours**		**12 hours**		**24 hours**	
	Hs (m)	**CI (±m)**	**Hs (m)**	**CI (±m)**	**Hs (m)**	**CI (±m)**	**Hs (m)**	**CI (±m)**	**Hs (m)**	**CI (±m)**
1	5.8	0.2	5.4	0.2	5.1	0.2	4.6	0.2	3.9	0.1
2	6.4	0.2	6.0	0.2	5.6	0.2	5.0	0.2	4.3	0.2
5	7.1	0.3	6.6	0.3	6.2	0.3	5.5	0.2	4.6	0.2
10	7.6	0.3	7.1	0.3	6.7	0.3	5.8	0.3	4.9	0.2
20	8.2	0.4	7.6	0.4	7.1	0.4	6.2	0.3	5.2	0.3
50	8.9	0.4	8.3	0.4	7.7	0.4	6.6	0.3	5.5	0.3
100	9.4	0.5	8.8	0.5	8.2	0.5	6.9	0.4	5.7	0.3

For most Waverider buoy stations, the calculated extreme significant wave heights have increased due to the addition of several major storms occurring since the analysis of 2010. Extreme value analysis is particularly influenced by the top 5 to 10 largest significant wave heights. Apart from Port Kembla, each Waverider buoy recorded at least one or two new storms within its “Top 5”. Coffs Harbour and Sydney values increased due to the measurement of new second and third largest significant wave heights at each buoy. Eden data analysis shows significantly higher values due to the record of new Top 5 largest storm wave heights.

Alternative Methods

Alternative extreme wave analysis approaches have been investigated and compared to the above results. These different approaches include:

- Annual Maximum Series using the following distributions:
  - Generalised Extreme Value (GEV) distribution including comparison between using the Maximum Likelihood Estimation (MLE), Bayesian Estimation (BE) and L-Moments Estimation (LME)
  - Gumbel distribution
- Alternative peaks-over-threshold approach using the Generalised Pareto (GP) distribution

All alternative methods have been calculated using the software environment for statistical computing and graphics named R. The comparison was completed using the Byron Bay Waverider Buoy dataset for a 1-hour duration storm.

Annual Maximum Series Method

The annual maximum series (AMS) method consists of determining the largest significant wave height that occurred each year of record. For longer storm durations, a moving minimum was applied to determine the storm wave heights. For example, for a 3-hour duration storm, a series including the minimum of the wave height of every 3 consecutive hours was created (which is equivalent to the wave height that is exceeded over the 3-hour duration) and was applied to the data. For each year, the maximum value of this moving average was selected as the maximum 3-hour duration. Once the 40 yearly maximum wave heights identified, various statistical distribution were applied to the obtained dataset and the results are provided below.

GEV Distribution

Generalised Extreme Value distribution includes the Weibull, Fréchet and Gumbel distribution family and is presented below:

$$F(x;\mu,\sigma,\xi) = exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}$$

with: μ the location parameter
σ the scale parameter
ξ the shape parameter

The fitting of the distribution was completed using the following estimation method:

- Maximum Likelihood Estimation (MLE): this method consists of selecting the distribution the most consistent with the data. This method determines the distribution parameters that have the greater possibility of getting the observed data than any other choice of parameters. The fitted GEV distribution using MLE is illustrated in Figure 4.
- L-moments Estimation: L-moments are summary statistics for probability distributions and data samples. They are analogous to ordinary moments and provide measures of location, dispersion, skewness and other aspects of the shape of probability distributions. However, they are computed from linear combinations of the ordered data values. The fitted GEV distribution using L-moments is illustrated in Figure 5.
- Bayesian Estimation: This method consists of assuming that an unknown parameter is known to have a prior distribution and considers the loss function based on some measurements. The Bayesian estimator is the estimator that minimises the posterior expected loss for each measurement. The fitted GEV distribution using Bayesian Estimation is illustrated in Figure 6.

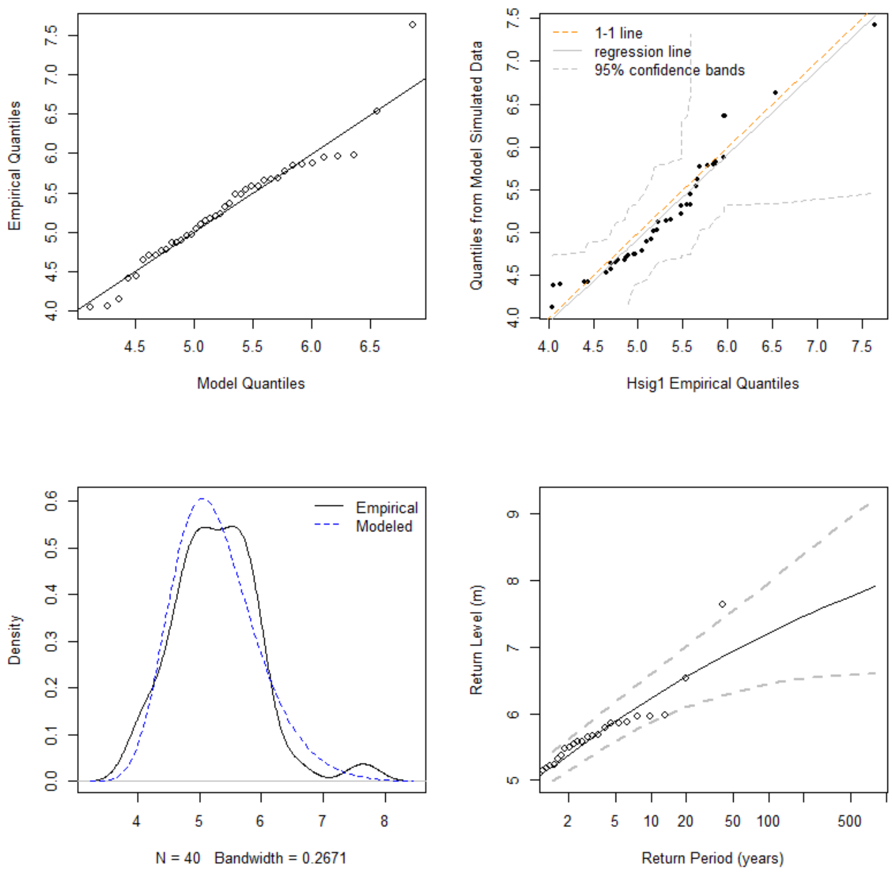

Figure 4 – GEV Distribution with MLE Estimation Results (Byron Bay, 1-hour Duration)

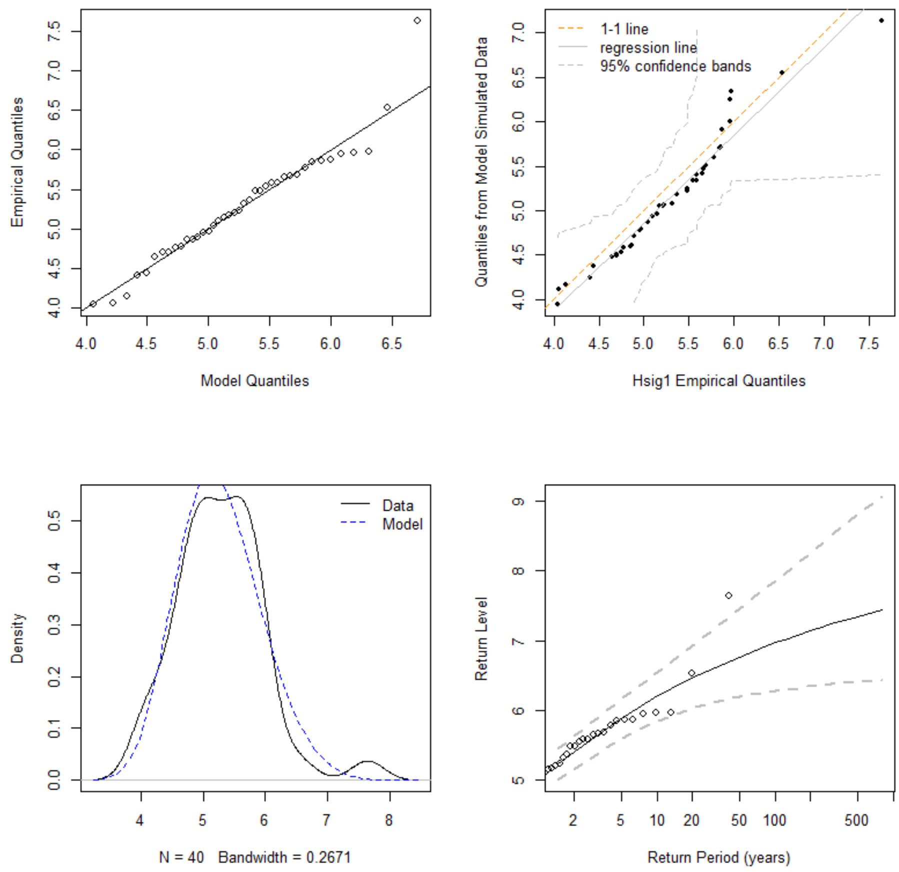

Figure 5 – GEV Distribution with L-Moments Estimation Results (Byron Bay, 1-hour Duration)

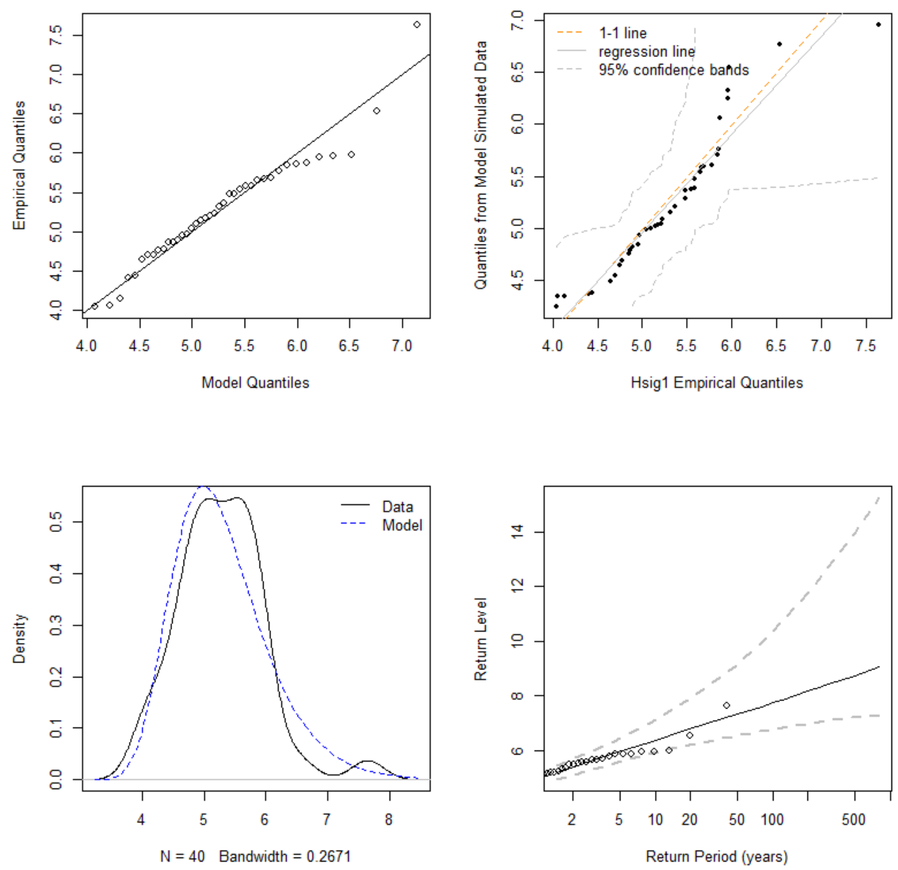

Figure 6 – GEV Distribution with Bayesian Estimation Results (Byron Bay, 1-hour Duration)

Gumbel Distribution

The Gumbel distribution is part of the GEV sub-families with $\xi = 0$ and the formula is presented below:

$$F(x;\mu,\sigma,0) = exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right]$$

The fitted Gumbel distribution is illustrated in Figure 7.

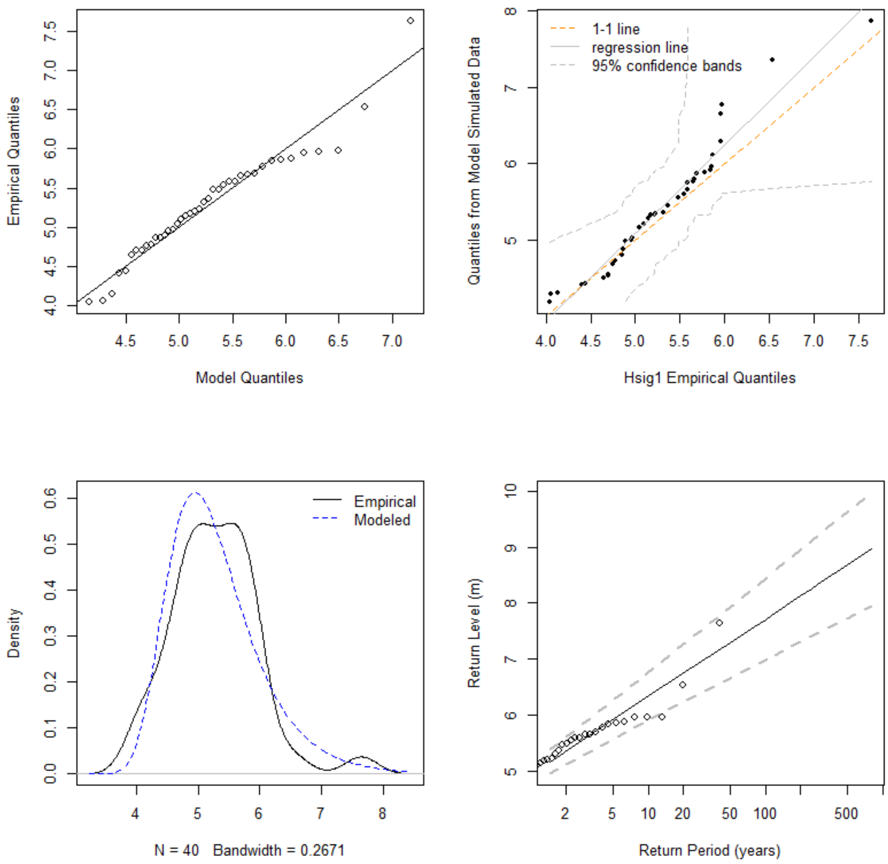

Figure 7 – Gumbel Distribution with MLE Estimation Results (Byron Bay, 1-hour Duration)

Alternative Peak-Over-Threshold Method

The alternative peak-over-threshold method used is fitting of the Generalised Pareto (GP) distribution on the same data set. The GP distribution is an approximation of the upper tail of a parent distribution function and this distribution is presented below:

$$H(x) = 1 - \left[1 + \xi\left(\frac{x-\mu}{\sigma_u}\right)\right]_+^{-1/\xi}$$

Where u is a high threshold (here 3m significant wave height), x>u and scale parameter σ_u depends on the threshold u. The fitted GP distribution is illustrated in Figure 8.

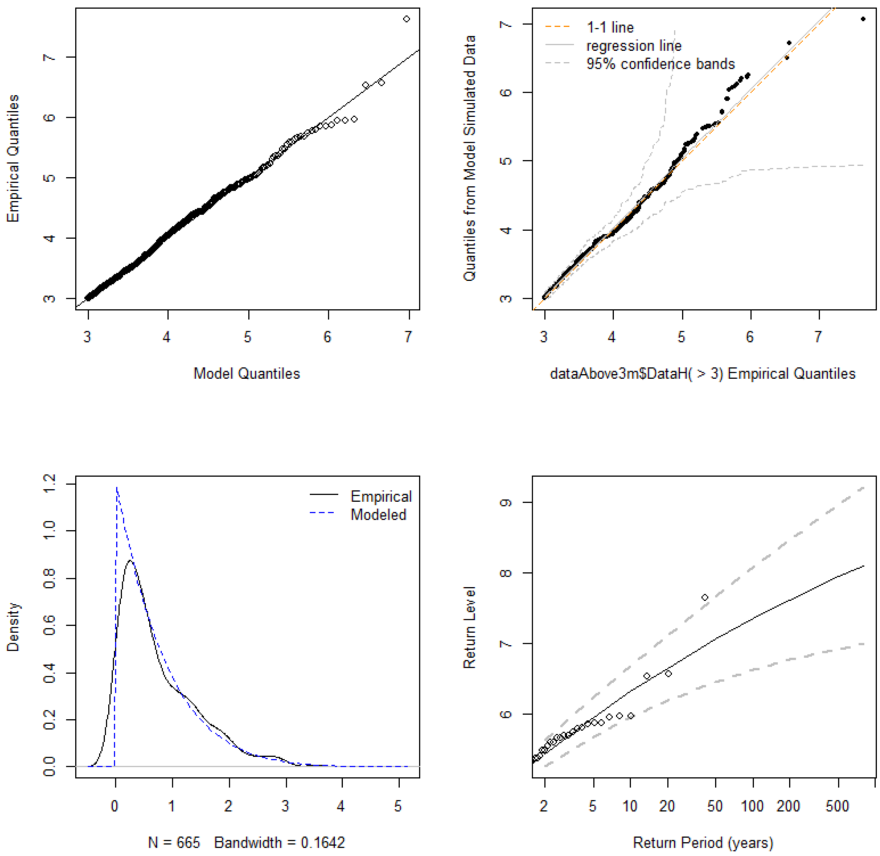

Figure 8 – Generalised Pareto Distribution (Byron Bay, 1-hour Duration)

Results Comparison

Figures 4 to 8 present the results of each of the alternative solutions described above. These figures include the quantile-quantile plot (top left), quantiles from a sample drawn from the fitted distribution against the empirical data quantiles with 95% confidence bands (top right), density plots of empirical data and fitted distribution (bottom left), and return level plot with 95% normal approximation confidence intervals (bottom right).

The scale, location and shape parameters for each distribution (including the original distribution used for the extreme wave analysis of each Waverider buoy) are provided in Table 13. The wave height for each distribution for return periods ranging between 2 year and 100 year ARI and associated confidence intervals are presented in Table 14.

It can be noted that all the different distributions and estimation method analysed are providing similar results. The adopted Weibull distribution provides slightly higher results than the other distribution which is adequate as it represents a conservative approach for engineering purposes.

This method was applied to the other storm durations and similar results were observed.

Table 13 – Scale, position and shape parameters for each function

Distribution	Parameters		
	Location	Scale	Shape
Weibull (Adopted)	2.920	0.882	-0.342
FT-I	3.441	0.536	-
GEV (MLE)	4.983	0.612	-0.106
GEV (L-Moments)	5.005	0.635	-0.186
GEV (Bayesian)	4.955	0.648	-0.051
Gumbel	4.948	0.601	-
GP	-	0.825	-0.098

Table 14 – Return periods and associated confidence intervals

ARI (yr)	Statistical Distribution													
	Weibull (Adopted)		FT-I		GEV (MLE)		GEV (L-moments)		GEV (Bayesian)		Gumbel		GP	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
2	5.7	0.2	5.5	0.2	5.2	0.2	5.2	0.2	5.2	0.2	5.2	0.2	5.4	0.2
5	6.2	0.2	6.0	0.2	5.8	0.3	5.8	0.2	5.9	0.3	5.8	0.3	6.0	0.3
10	6.6	0.2	6.3	0.2	6.2	0.4	6.2	0.3	6.3	0.5	6.3	0.4	6.3	0.4
20	7.0	0.3	6.7	0.3	6.5	0.4	6.5	0.4	6.8	0.8	6.7	0.5	6.6	0.5
50	7.5	0.3	7.2	0.3	6.9	0.6	6.8	0.5	7.3	1.3	7.3	0.6	7.1	0.6
100	7.8	0.3	7.6	0.3	7.2	0.8	7.0	0.7	7.7	1.8	7.7	0.7	7.3	0.7

Directional Analysis

Preliminary wave directional analysis was completed as part of this study. Table 1 presents a summary of the record lengths for each buoy. Three Waverider stations have recorded over 15 years of directional data while the other stations have less than six years of directional data. The directional analysis therefore focused on the three longer duration data sets (i.e. Sydney, Byron Bay and Batemans Bay) that provide more relevant information than the shorter data sets for the directional extreme value analysis. Figure 9 presents wave roses (H_s and direction) for each Waverider buoy station. The number of storms measured in each direction for the three selected Waverider buoys is provided in Table 16.

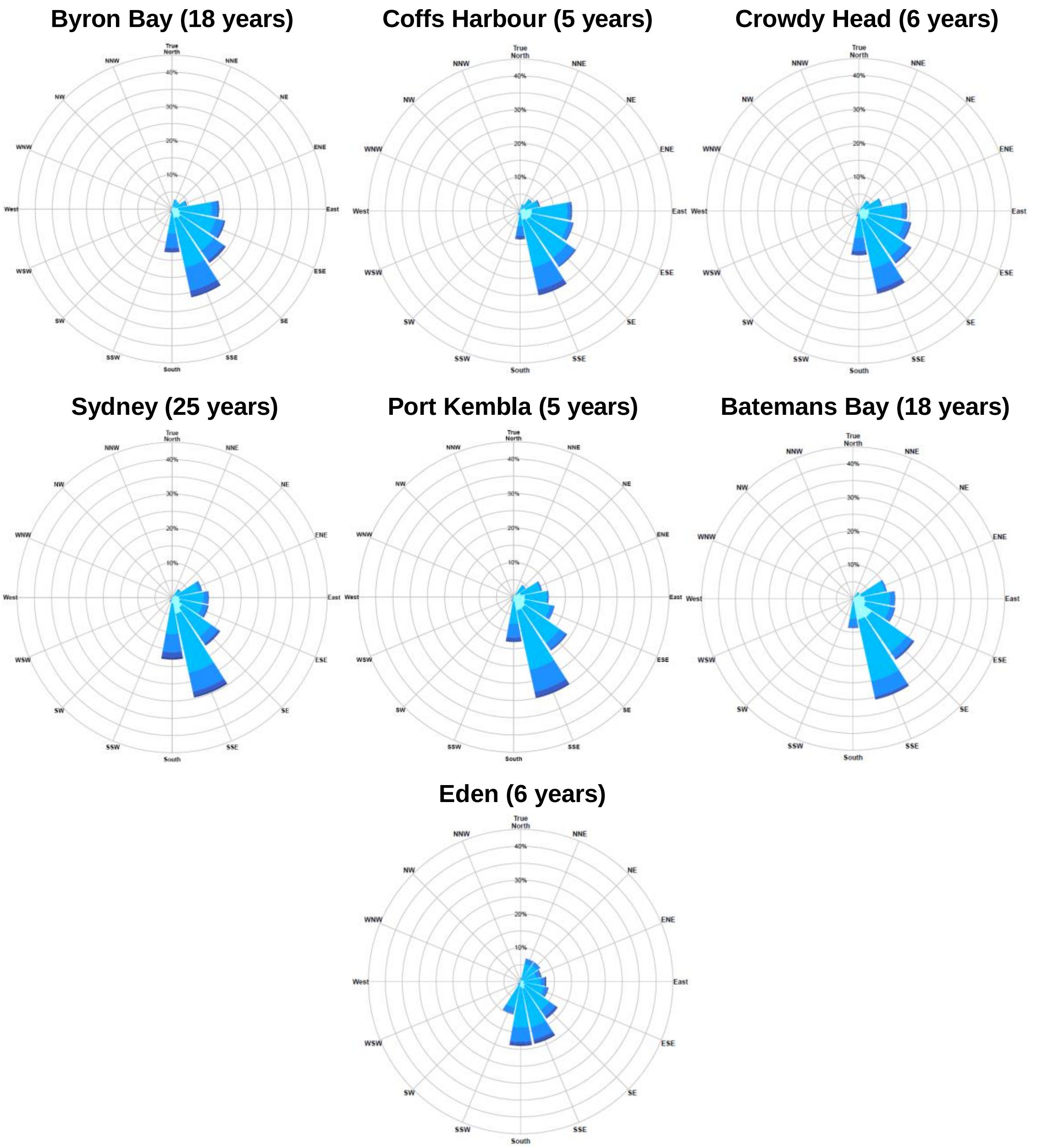

Figure 9 – Wave roses at all sites

Table 16 – Number of storms per direction

Direction*	Number of Storms per Direction		
	Byron Bay	Sydney	Batemans Bay
NE	6	3	3
ENE	9	19	15
E	40	30	26
ESE	40	34	39
SE	78	63	59
SSE	172	186	159
S	186	338	144
SSW	9	14	3
TOTAL	612**	687	448

*Directions not included do not have any recorded storm event (i.e. event with $H_s \geq 3m$)
**A number of storms have been recorded without directional information for Byron Bay

A directional extreme value analysis was carried out using the storms from Table 16 and the results are provided in Figure 10. The following observations are noted:

- It can be seen that for north-east and south-west wave directions, the lack of recorded data typically affects the shape of the curve and therefore is not suitable for an EVA.
- Directions from south to east-south-east typically have a similar trend to the overall curve due to the larger number of storm events that originate from these directions.
- The east direction from Byron Bay has been significantly influenced by the largest storm recorded in May 2009 which steepens the resulting curve.

A summary of the results of the directional EVA is presented in Table 17. Figures in grey italic in this table represent erroneous results due to lack of data or skewed plotting values.

Given the short duration of these data sets of 15 to 25 years and the low number of recorded storm events, it is important to note that the values calculated for the below ARIs are extrapolated and may be inappropriate for use for design purposes.

Table 17 – Directional extreme value analysis results

ARI (yr)	Statistical Distribution (Weibull)															
	NE		ENE		E		ESE		SE		SSE		S		SW	
	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)	Hs (m)	CI (±m)
Byron Bay																
2	*N/A*	*N/A*	*N/A*	*N/A*	*4.4*	*0.2*	4.4	0.2	4.5	0.2	4.7	0.1	4.8	0.1	*N/A*	*N/A*
5	*3.1*	*0.4*	*3.6*	*0.2*	*5.4*	*0.4*	5.0	0.2	5.1	0.2	5.3	0.2	5.2	0.1	*3.5*	*N/A*
10	*3.9*	*0.6*	*3.9*	*0.2*	*6.1*	*0.5*	5.4	0.3	5.5	0.3	5.6	0.2	5.6	0.2	*3.8*	*N/A*
20	*4.7*	*1.0*	*4.2*	*0.3*	*6.8*	*0.6*	5.8	0.3	6.0	0.3	6.0	0.2	5.9	0.2	*4.1*	*N/A*
50	*5.6*	*1.5*	*4.5*	*0.4*	*7.7*	*0.8*	6.3	0.4	6.5	0.4	6.4	0.3	6.2	0.2	*4.4*	*N/A*
100	*6.2*	*1.8*	*4.8*	*0.5*	*8.4*	*0.9*	6.7	0.4	6.9	0.4	6.8	0.3	6.5	0.2	*4.5*	*N/A*
Sydney																
2	*N/A*	*N/A*	3.5	0.2	3.8	0.2	3.8	0.2	4.8	0.3	5.5	0.2	5.7	0.2	*3.0*	*0.2*
5	*N/A*	*N/A*	4.2	0.3	4.6	0.4	4.6	0.4	5.7	0.4	6.4	0.3	6.4	0.2	*3.7*	*0.2*
10	*3.3*	*0.5*	4.6	0.4	5.2	0.5	5.3	0.6	6.4	0.5	7.0	0.4	6.9	0.2	*4.1*	*0.3*
20	*3.8*	*1.1*	5.1	0.5	5.8	0.6	6.0	0.7	7.1	0.6	7.6	0.4	7.4	0.3	*4.5*	*0.5*
50	*4.5*	*2.0*	5.6	0.6	6.5	0.8	6.8	1.0	8.0	0.7	8.5	0.5	8.0	0.3	*5.0*	*0.6*
100	*5.0*	*2.7*	6.0	0.7	7.1	0.9	7.5	1.1	8.6	0.8	9.1	0.6	8.5	0.3	*5.3*	*0.7*
Batemans Bay																
2	*N/A*	*N/A*	*3.0*	*0.2*	3.6	0.3	4.1	0.2	4.1	0.2	4.8	0.2	4.4	0.1	*N/A*	*N/A*
5	*N/A*	*N/A*	*3.6*	*0.2*	4.6	0.5	4.9	0.3	4.7	0.2	5.4	0.2	4.8	0.1	*N/A*	*N/A*
10	*3.1*	*0.3*	*4.0*	*0.3*	5.3	0.8	5.5	0.4	5.1	0.3	5.8	0.3	5.1	0.2	*N/A*	*N/A*
20	*3.4*	*1.2*	*4.4*	*0.5*	6.2	1.1	6.1	0.5	5.4	0.3	6.3	0.3	5.4	0.2	*N/A*	*N/A*
50	*4.0*	*2.9*	*4.9*	*0.7*	7.4	1.5	6.8	0.7	5.9	0.4	6.8	0.3	5.7	0.2	*N/A*	*N/A*
100	*4.5*	*4.4*	*5.3*	*0.8*	8.3	1.8	7.3	0.8	6.2	0.4	7.2	0.4	6.0	0.3	*N/A*	*N/A*

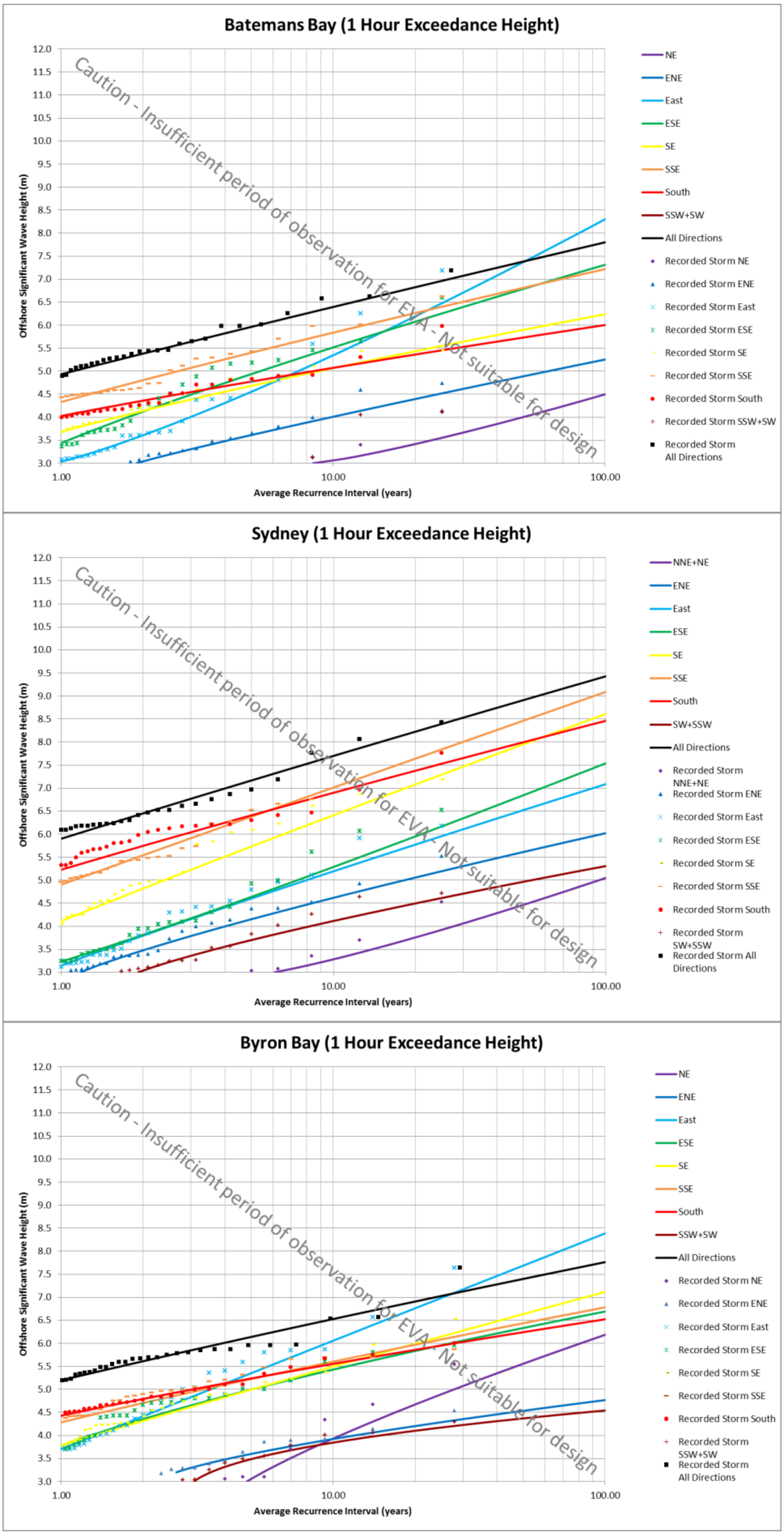

Figure 10 – Directional Extreme Wave Analysis Results

Conclusion

Regular update of the extreme value analysis is important to maintain an up-to-date understanding of the occurrence probabilities of large events. Especially given the current uncertainties generated by climate change and its impact on storminess. Major events are defining the tail of the distribution and recent significant storms such as the 5-6 June 2016 can have major influence on the results of the analysis as highlighted by the changes in the results for the Eden Waverider buoy station.

Various statistical distributions were assessed and were found to result in similar values. The Weibull distribution was selected as it gives slightly larger results providing a more conservative approach for coastal engineering purposes.

Directional extreme wave analysis completed as part of this study highlighted that the various data sets were too short to provide reliable results with some buoys only recording three storms over the record period for some directions (particularly in the N-NE and S-SW quadrants). It is therefore not recommended to use these data for coastal engineering and design purposes. However, it provides an indication of the relative difference of results between the different directions. It is important to note that less frequent directions may well be characterised by the largest of all extreme wave heights and these may simply have not yet occurred during the directional wave recording period. Until the directional record length extends for a sufficient period of time, the possibility of extreme wave heights applicable to all directions should be considered by designers as potentially occurring from any incident offshore direction.

Hourly significant wave height data are currently discarded when some data are missing during the recorded time. Development of additional data recovery processes using the raw wave data would therefore be recommended to fill a number of gap in the existing historical data.

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